A Comparative Investigation of Rotation Criteria Within Exploratory Factor Analysis

Daniel A. Sass [a]; Thomas A. Schmitt [b]
[a] University of Texas at San Antonio, [b] Eastern Michigan University,

Online publication date: 25 February 2010

PLEASE SCROLL DOWN FOR ARTICLE

Ψ Psychology Press
Taylor & Francis Group

A Comparative Investigation of Rotation Criteria Within Exploratory Factor Analysis

Daniel A. Sass
University of Texas at San Antonio

Thomas A. Schmitt
Eastern Michigan University

Exploratory factor analysis (EFA) is a commonly used statistical technique for examining the relationships between variables (e.g., items) and the factors (e.g., latent traits) they depict. There are several decisions that must be made when using EFA, with one of the more important being choice of the rotation criterion. This selection can be arduous given the numerous rotation criteria available and the lack of research/literature that compares their function and utility. Historically, researchers have chosen rotation criteria based on whether or not factors are correlated and have failed to consider other important aspects of their data. This study reviews several rotation criteria, demonstrates how they may perform with different factor pattern structures, and highlights for researchers subtle but important differences between each rotation criterion. The choice of rotation criterion is critical to ensure researchers make informed decisions as to when different rotation criteria may or may not be appropriate. The results suggest that depending on the rotation criterion selected and the complexity of the factor pattern matrix, the interpretation of the interfactor correlations and factor pattern loadings can vary substantially. Implications and future directions are discussed.

Since Spearman (1904) first proposed the idea of a single common factor to explain "general intelligence," both exploratory factor analysis (EFA) and confirmatory factor analysis (CFA) have become widely used statistical procedures

---

Correspondence concerning this article should be addressed to Daniel A. Sass, University of Texas at San Antonio, Department of Educational Psychology, 501 West Durango Blvd., DB 4.322, San Antonio, TX 78207. E-mail: daniel.sass@utsa.edu

during instrument development (see Gorsuch, 1983). Even though EFA and CFA are both based on the common factor model and share the same goal of explaining the manifest variable's covariances or correlations, the appropriate use of EFA remains controversial (Steiger, 1979, 1994) because it is driven by data, not theory. Despite the dilemma of whether EFA or CFA is more appropriate for examining relationships among indicators, EFA remains popular among researchers. In fact, this procedure has experienced a revival resulting from new EFA applications (e.g., Asparouhov & Muthén, 2009) and the realization that CFA can be restrictive and is often itself used in an exploratory fashion (Asparouhov & Muthén, 2009; Browne, 2001; Gorsuch, 1983; MacCallum, Roznowski, & Necowitz, 1992). For instance, CFA users frequently make the rather limited assumption that each variable is a pure measure (i.e., all cross-loadings are zero) of each factor. Consequently, they proceed to use modification indices in an exploratory fashion to improve model fit, which can result in a model fitting by chance (MacCallum et al., 1992). In any event, both EFA and CFA can be difficult statistical methods to employ correctly given the subjective decisions that must be made (see Henson & Roberts, 2006).

Fortunately, there have been several studies to help guide researchers through the factor analysis decision-making process. Such choices include selecting (a) an appropriate sample size (e.g., Hogarty, Hines, Kromrey, Ferron, & Mumford, 2005; MacCallum, Widaman, Zhang, & Hong, 1999), (b) the best model fitting/estimation procedure (e.g., principal factors, maximum likelihood, etc.; Flora & Curran, 2004), (c) a method to determine the number of factors (e.g., Kaiser criterion, parallel analysis, etc.; Hayton, Allen, & Scarpello, 2004), (d) the proper correlation matrix (e.g., Pearson, polychoric, tetrachoric, etc.; Jöreskog & Moustaki, 2001), and (e) a rotation criterion (Varimax, Promax, etc.; Browne, 2001). To supplement these papers, several comprehensive review articles have also been written to guide researchers using EFA (e.g., Fabrigar, Wegener, MacCallum, & Strahan, 1999; Henson & Roberts, 2006).

Despite the attention each of these areas has received, a paucity of literature has (a) investigated how rotation criteria differ under diverse factor pattern matrices, (b) discussed the implications for choosing different rotation criteria in relation to factor structure interpretation, and (c) provided guidance for selecting an appropriate rotation criterion depending on the perceived variable or factor pattern matrix complexity. With the exception of Browne's (2001) comprehensive paper, few recent articles have outlined, compared, and evaluated different rotation criteria. In our review of the literature, and in the opinion of other authors (Browne, 2001; Fabrigar et al., 1999; Finch, 2006; Henson & Roberts, 2006; Thompson & Daniel, 1996), rotation criteria are largely selected based on whether or not the hypothesized factors are orthogonal or oblique, with little consideration given to potential factor structure complexity. Despite the importance of selecting a rotation criterion based on interfactor correlations, it

is perhaps as important to consider the influence different rotation criteria will have on the resultant factor pattern matrix.

Although researchers often select either an orthogonal or an oblique rotation criterion based on hypothesized interfactor correlations, many are unacquainted with how common rotation criteria differ. For example, if an oblique rotation criterion were selected based on either prior research or theory, many researchers would give little justification for its selection (Henson & Roberts, 2006; e.g., Promax, Quartimin, Equamax, etc.) or how the selected rotation criterion influences factor structure interpretation (see de Vet, Adèr, Terwee, & Pouwer, 2005). Instead, researchers often select a rotation criteria based on what is most frequently cited in the literature, which is often an orthogonal Varimax criterion (Fabrigar et al., 1999; Ford, MacCallum, & Tait, 1986; Henson & Roberts, 2006; Russell, 2002). For example, when using EFA and CFA for cross validation studies researchers regularly assume factor structure differences are due to sampling error or sample characteristics (Hurley et al., 1997), with less attention given to the estimation methods, correlation matrix employed, and/or the rotation criteria. It is important that researchers are cognizant that the rotation criterion selected could have a significant impact on the interfactor correlations and the cross-loading magnitudes.

FACTOR STRUCTURE TERMINOLOGY

This study defines variable complexity as the number of nonzero elements in a factor pattern matrix row (see Browne, 2001; Jennrich, 2007). A factor pattern matrix where all variable complexities are one results in *perfect simple structure*, which means that each variable loads on only one factor and no cross-loadings exist. When the cross-loading magnitudes are small (i.e., $< |.30|$), this is considered *approximate simple structure*. As the cross-loading magnitudes increase (i.e., $\geq |.30|$), the factor structure is referred to as *complex structure*.

Despite the delineation of aforementioned different factor pattern matrices, the criterion generally accepted to provide a meaningful rotated factor solution is the principle of *simple structure* (Thurstone, 1947). Simple structure seeks a parsimonious solution in the primary factor pattern matrix by placing a general set of restrictions on the factor analysis model. Although these constraints placed on the factor analysis model do not affect the model fit to the observed data, they do have significant implications for the interpretation of the rotated factor pattern matrix (Yates, 1987). Thus, the simple structure criterion solves the indeterminacy problem in EFA by placing a set of restrictions on the factor analysis model while also producing meaningful solutions (Gorsuch, 1983, p. 177).

The idea of simple structure was originally proposed by Thurstone (1947, p. 335), who provided five general conditions for its evaluation:

1. Each variable (e.g., row) should have at least one zero.
2. Each factor (e.g., column) should have the same number of zeros as there are factors.
3. Every pair of factors should have several variables whose factor pattern loadings are zero for one factor but not the other.
4. Whenever more than about four factors are extracted, every pair of factors should have a large portion of variables with zero weights in both factors.
5. Every pair of factors should have few variables with nonzero pattern loadings in both factors.

It was the first condition that Thurstone (1947) intended to encompass his definition of simple structure, a definition that allowed for more complex factor pattern matrices (Browne, 2001; Jennrich, 2007; McDonald, 2005; Yates, 1987). Thurstone did not envision conditions two through five as defining simple structure, but only as checks of uniqueness and stability of the factor pattern loading configurations (Browne, 2001; Yates, 1987). However, many early factor analysts developed simple structure rotation criteria, such as Kaiser's (1958) Varimax criterion, with the intention of seeking perfect cluster configurations or ones that were easily interpretable. The difficulty with such an interpretation is that many measures/instruments have complex factor loading patterns.

The concept of *complexity* (Browne, 2001; Yates, 1987) is central to EFA because it encompasses Thurstone's (1947) notion of simple structure in that factor rotation criteria should allow factorially complex solutions and not just perfect independent cluster solutions (i.e., perfect simple structure). Further, allowing for solutions that are more complex provides a more realistic depiction of the domains of interest. As Yates pointed out, many popular and commonly used rotation criteria (e.g., Varimax) adhere to the somewhat misguided notion that simple structure criterion should seek perfect independent clusters that are easily interpreted, which consequently may not represent the factor of interest.

Unfortunately, researchers are often enamored with simple structure because of its clean and easy-to-interpret solutions, and frequently fail to realize that important insight gained from more complex and realistic factor structures (Browne, 2001). For example, take a two-factor test measuring math and reading. Using a rotation criterion that seeks perfect simple structure, or minimum row complexity, may give a factor structure with smaller cross-loadings and a larger interfactor correlation, whereas a rotation criterion that allows greater row complexity will provide evidence of items that measure both reading and math and reduce the interfactor correlation. Consequently, the detection and removal of items that assess reading and math will not only purify the factors but will also reduce the interfactor correlation and increase discriminate validity. Employing a rotation criterion that reveals the complexity of the factor structure can have

considerable benefits during instrument construction. Because the relationships between variables and factors are often complex, it is imperative that rotation criteria extract pattern matrices that accurately depict these complexities.

INTERFACTOR CORRELATIONS

Fabrigar et al. (1999) emphasized that many factors are correlated, and therefore oblique rotations more accurately and realistically depict the "true factor structure." Harman (1976) further stated that if factors are orthogonal, a successful oblique rotation would accurately estimate interfactor correlations near zero and arrive at solutions close to those produced by an orthogonal rotation. Ideally, oblique rotation criteria will provide valid solutions for factors' structures that have either correlated or uncorrelated factors and therefore provide a more flexible analytic approach. Although often forgotten, Gorsuch (1983, p. 71) recommended factor analysis not be employed with interfactor correlations greater than .50. This consideration is significant given that subscales are often highly correlated within a measure.

Analytic Rotation Criteria

Analytic rotation criteria are based on clearly defined rules that generally require little to no input from the researcher. This study reviews seven analytic rotation criteria (Crawford-Ferguson, Quartimin, CF-Varimax, CF-Equamax, CF-Parsimax, CF-Facparsim, & Geomin) available in Mplus Version 5.2 (Muthén & Muthén, 1998–2007). Mplus was selected given its inclusion of numerous rotation criteria and its ability to specify various correlation matrices and estimation methods.

For the following rotation criteria, Λ is defined as a factor pattern loading matrix with p rows (i.e., variables) and m columns (i.e., factors), and $f(\Lambda)$ is a continuous function of factor pattern loadings that measures the factor pattern loading complexity in Λ (i.e., complexity function). The goal for analytic rotation criteria is to minimize the complexity function $f(\Lambda)$ so that the rotated factor pattern matrix Λ has a pattern of simple or interpretable pattern loadings (see Browne, 2001, for more detailed factor analysis notation). To supplement the description of each rotation, Table 1 provides a summary of each rotation criteria discussed.

Most of the rotation criteria considered here fit into the Crawford-Ferguson (CF) family (Crawford & Ferguson, 1970). CF rotation criteria use a weighted sum of the row (variable) and column (factor) complexity measures and reflects Thurstone's (1947) conditions two through five (Browne, 2001). The CF family

TABLE 1
A Description of Rotation Criteria

Name	Principle for Guiding Rotation	Relationship to Crawfer	Comment
Crawfer	Minimizes variable and factor complexity based on k	Not applicable	Smaller k minimizes variable complexity and larger k minimizes factor complexity.
Quartimin	Minimizes variable complexity	$k = 0$	Developed to variable complexity; works well with distinct clusters (i.e., no cross-loadings).
CF-Varimax	Minimizes variable and factor complexity based on p variables	$k = \dfrac{1}{p}$	Developed to spread variances across factors; larger p minimizes variable complexity and smaller p minimizes factor complexity.
CF-Equamax	Minimizes variable and factor complexity based on p variables and m factors	$k = \dfrac{m}{2p}$	Like CF-Varimax, developed to spread variances more equally across factors.
CF-Parsimax	Minimizes variable and factor complexity based on p variables and m factors	$k = \dfrac{m-1}{p+m-2}$	Developed to equally minimize variable and factor complexity.
CF-Facparsim	Minimizes factor complexity	$k = 1$	Developed to minimize factor complexity.
Geomin	Minimizes variable complexity	Not applicable	Developed to minimize variable complexity to more adequately represent Thurstone's (1947) conceptualization of simple structure, which allows for greater variable complexity than Quartimin; $\in$ reduces indeterminacy; will often produce results similar to Quartimin.

follows the following form:

$$f(\Lambda) = (1-k) \underbrace{\sum_{i=1}^{p}\sum_{j=1}^{m}\sum_{l \neq j, l=1}^{m} \lambda_{ij}^2 \lambda_{il}^2}_{\text{Row complexity}} + k \underbrace{\sum_{j=1}^{m}\sum_{i=1}^{p}\sum_{l \neq i, l=1}^{p} \lambda_{ij}^2 \lambda_{lj}^2}_{\text{Column complexity}}. \qquad (1)$$

The CF criterion, also referred to as the Crawfer rotation criterion, is indexed by a single parameter, $k(0 \leq k \leq 1)$. Larger values of k place more weight on column complexity, whereas smaller k values put greater emphasis on row complexity. The Crawfer rotation has appeal because the user can specify a complexity function (i.e., rotation criterion) by changing the value of k. A value of $k = 1$ (i.e., CF-Facparsim) puts all the weight on column/factor complexity.

Thus, the row complexity term is dropped from the equation and only column complexity is minimized, allowing for greater cross-loadings. This is reflective of Thurstone's (1947) second condition, which states there must be at least the same number of zeros in each column as there are factors. Rotation criteria attempting to minimize only column complexity may be less likely to produce a simple structure when compared with rotations that minimize row complexity.

Setting $k = 0$ (i.e., Quartimin) places more weight on row/variable complexity and seeks to obtain a perfect cluster configuration (Carroll, 1953). Essentially, $k = 0$ seeks a solution closer to perfect simple structure as the number of zeros for each row/variable is maximized. This criterion works well when distinct clusters exist, but it may overemphasize the reduction of row complexity resulting in unrealistically high interfactor correlations (Carroll, 1953; Gorsuch, 1983; Yates, 1987). It is important to remember that Thurstone (1947) did not intend for rotation criteria to obtain factor pattern matrices with only zero or near zero cross-loadings, but instead interpretable solutions that best represent the hypothesized factor pattern matrix.

In addition to the Crawfer rotation described earlier, several other rotations (Quartimin, CF-Varimax, CF-Equamax, CF-Parsimax, CF-Facparsim, & Geomin) are worth noting because they could produce different factor pattern matrices depending on the values of p and m. This difference is based on a slight variation in k when changing the number of variables and factors.

Direct Quartimin ($k = 0$).   The rotation criterion is

$$f(\Lambda) = \sum_{i=1}^{p} \sum_{j=1}^{m} \sum_{l \neq j}^{m} \lambda_{ij}^2 \lambda_{il}^2.$$

(2)

The Direct Quartimin criterion, referred to as Quartimin within Mplus, measures row (i.e., variable) complexity and encompasses Thurstone's (1947) third, fourth, and fifth conditions (Browne, 2001). If distinct clusters of variables exist within the data (i.e., each row in the pattern loading matrix, Λ, has only a single nonzero loading), then generally no other rotation criterion will perform better (Asparouhov & Muthén, 2009; Yates, 1987). Direct Quartimin was developed with the idea that rotating to perfect simple structure is done solely to ensure that clusters of variables are independent and easily interpretable. However, many variables are not perfect measures of each factor, and therefore can result in greater factorial complexity (e.g., cross-loadings). From an interpretability standpoint, variables may appear correlated with only one factor, even though these variables in theory are correlated with other factors. It should be noted that the Direct Quartimin criterion is equal to the CF-Quartimax criterion when rotated obliquely.

CF-Varimax (k = 1/p).   The rotation criterion is

$$f(\Lambda) = \left(1 - \frac{1}{p}\right) \sum_{i=1}^{p} \sum_{j=1}^{m} \sum_{l \neq j}^{m} \lambda_{ij}^2 \lambda_{il}^2 + \frac{1}{p} \sum_{i=1}^{m} \sum_{i=1}^{p} \sum_{l \neq i}^{p} \lambda_{ij}^2 \lambda_{lj}^2. \tag{3}$$

Varimax (Kaiser, 1958) is essentially a variation on the Direct Quartimin criterion and works to spread the variance across factors to prevent the occurrence of a single factor (Yates, 1987). As can be seen in Equation 3, when the number of variables of (p) increases, the value of k ($k = 1/p$) gets smaller, placing more emphasis on row/variable complexity than column/factor complexity. It is worth noting that CF-Varimax and Varimax will produce identical solutions with orthogonal factors. However, oblique Varimax can result in factor collapse, meaning that correlations between factors have a tendency to approach one. This does not occur for the CF-Varimax rotation criterion or any other member of the CF family (Browne, 2001; Crawford, 1975). For the aforementioned reasons, it will be interesting to investigate how CF-Varimax performs with complex data structures and highly correlated factors.

CF-Equamax (k = m/2p).   The rotation criterion is

$$f(\Lambda) = \left(1 - \frac{m}{2p}\right) \sum_{i=1}^{p} \sum_{j=1}^{m} \sum_{l \neq j}^{m} \lambda_{ij}^2 \lambda_{il}^2 + \frac{m}{2p} \sum_{i=1}^{m} \sum_{i=1}^{p} \sum_{l \neq i}^{p} \lambda_{ij}^2 \lambda_{lj}^2. \tag{4}$$

Similar to Equamax, CF-Equamax combines Quartimax and Varimax criteria by simplifying both the variables and factors in the factor pattern matrix and spreading variances more equally across the factors (Gorsuch, 1983). Unlike CF-Varimax, CF-Equamax computes k ($k = m/2p$) based on the number of variables and factors. Thus, this ratio needs to be considered when understanding whether CF-Equamax is aimed more at variable or factor complexity. Similar to the other CF procedures, researchers need to be mindful that the function (meaning seeking more variable or factor complexity) of each rotation criterion can change based on p and m.

CF-Parsimax (k = m − 1/p + m − 2).   The rotation criterion is

$$f(\Lambda) = \left(1 - \frac{m-1}{p+m-2}\right) \sum_{i=1}^{p} \sum_{j=1}^{m} \sum_{l \neq j}^{m} \lambda_{ij}^2 \lambda_{il}^2 + \frac{m-1}{p+m-2} \sum_{i=1}^{m} \sum_{i=1}^{p} \sum_{l \neq i}^{p} \lambda_{ij}^2 \lambda_{lj}^2. \tag{5}$$

The CF-Parsimax criterion was proposed by Crawford and Ferguson (1970) so that equal weight was given to both variable and factor complexity. Crawford

and Ferguson found the Parsimax criterion to be as good as Equamax but found it sensitive to the number of factors. They also suggested using it when the number of factors is reasonably known.

CF-Facparsim (k = 1).   The rotation criterion is

$$f(\Lambda) = \sum_{j=1}^{m} \sum_{i=1}^{p} \sum_{l \neq i}^{p} \lambda_{ij}^2 \lambda_{lj}^2. \tag{6}$$

As opposed to the Direct Quartimin criterion, which attempts to minimize variable complexity, CF-Facparsim seeks to minimize only factor complexity. Because CF-Facparsim consists only of the factor complexity expression from Equation 1, Browne (2001) argued it was more of theoretical interest. Crawford and Ferguson (1970) also pointed out that, like CF-Equamax, CF-Facparsim spreads variances equally across all rotated factors and should only be used when researchers have a strong hypothesis for the number of factors. Nevertheless, this rotation will be considered given that it encompasses the main goal of EFA, which as Gorsuch (1983) stated, "is to simplify a factor rather than a particular variable because the interest invariably lies in learning more about the factors rather than the variables" (p. 184).

Yates's Geomin.   The Geomin criterion was developed as a compromise between Thurstone's (1947) condition one and conditions four and five. Recall that the first condition allows for a more complex factor pattern matrix as variables can load on multiple factors, which was Thurstone's intended definition for simple structure. Conditions four and five demand that variables load primarily on a single factor, provided an interpretable solution exists. Yates (1987) thought it was important to follow Thurstone's desire to accurately depict complex factor loading matrices yet also provide interpretable solutions. Consequently, Yates developed the Geomin rotation criterion with the idea that a good rotation criterion would allow for complex factors and provide an interpretable pattern matrix. The complexity function used within Mplus is a slight modification of Yates's equation as a small positive value ($\in$) was added to alleviate indeterminacy. This equation, which includes $\in$, was proposed by Browne (2001):

$$f(\Lambda) = \sum_{i=1}^{p} \left[\prod_{j=1}^{m} (\lambda_{ij}^2 + \in) \right]^{\frac{1}{m}}. \tag{7}$$

This small change reduces indeterminacy and, for the most part, does not affect $f(\Lambda)$ if $\in$ values remain small. For example, if $\in$ is zero or very small (e.g.,

10^{-6}) and one pattern loading for each variable is zero, then the Geomin criterion function is minimized and it does not help in the identification of the remaining factor pattern loadings. Browne (2001) noted that $\in \, = .01$ works well for three or four factors but may need to be increased for more than four factors. The Geomin rotation criterion can be used for orthogonal rotations, but Yates (1987) did not intend it for such data structures. The Geomin and Quartimin criteria should produce comparable solutions, especially under pattern loading structures with a negligible number of small cross-loadings.

Although Browne's (2001) article was instrumental in proposing how different rotation criteria can influence factor pattern matrices, few empirical studies exist that comprehensively evaluate the various rotation criteria. This study investigated and compared how different rotation criteria can produce conflicting factor pattern matrices and interfactor correlations. As a result, researchers can start to recognize, or at least be cognizant of, how their choice of a rotation criterion impacts factor structure interpretation. Moreover, this article seeks to help researchers better understand the various oblique rotation criteria along with the factor pattern loading structures they can expect to obtain under the studied conditions.

METHODS

Experimental Conditions

For this study, 128 experimental conditions were conducted to explore four factor structures (perfect simple structure, approximate simple structure, complex structure, and general structure), eight population interfactor correlations (ρ = .00, .10, .20, .30, 40, .50, .60, and .70), and four rotation criteria (Quartimin, $k = 0$; CF-Equamax, $k \approx .03$; CF-Facparsim, $k = 1$; and Geomin). Each experimental condition was replicated (R) 1,000 times to evaluate factor pattern loading bias and stability. In addition to the Monte Carlo simulation, rotated population results were provided to eliminate sampling error.

Sample Size

Previous literature reviews have indicated that most published research using EFA utilize sample sizes of roughly 300 (Fabrigar et al., 1999; Ford et al., 1986; Russell, 2002), with $n = 300$ also considered a "good" sample size according to Comrey and Lee (1992, pp. 216–217). Following standard practice, our Monte Carlo simulation study generated sample sizes of $n = 300$ to estimate the factor pattern loading matrices under the assumption that the correct model was specified. Past research (Hogarty et al., 2005; MacCallum et al., 1999) has

thoroughly evaluated the minimum sample size requirement for stable factor pattern loading estimates, therefore, different sample sizes were not considered. Instead, this study focused solely on how the rotation criterion, factor structure, and interfactor correlation influenced parameter recovery.

Data Generation

Data were generated in Mplus Version 5.2 using the true factor pattern loadings (λ_x & λ_y; see Table 2), interfactor correlation coefficients (i.e., ρ), and the variable/item residuals (ε_i). The variable/item residuals (ε_i) were calculated using the following equation: $\varepsilon_i = 1 - (\lambda_{i1}^2 + \lambda_{i2}^2 + 2(\lambda_{i1}^2)(\lambda_{i2}^2)(\rho))$, where λ_{i1}^2 and λ_{i2}^2 correspond to the factor pattern loadings for variable/item i on factors 1 and 2, respectively, and ρ is the interfactor correlation (see Gorsuch, 1983, pp. 29–30). Each of the variables/items were modeled as continuous, independent and identically distributed, linearly related to the factor, and with a standard normal distribution. Also note that the same data were analyzed across rotation criteria to ensure comparability.

Although tests/measures commonly possess either dichotomous or ordinal responses, data were not categorized to ensure that any estimation error was a direct result of the experimental conditions and not variable/item categorization. Moreover, the correlation matrix employed for ordinal data (i.e., polychoric correlation matrix) and the estimation method (i.e., weighted least squared mean and variance) would make it difficult to ascertain whether estimation bias resulted from the categorization process, correlation matrix employed, and/or estimation method.

The correlation matrices from the simulated data were analyzed using maximum likelihood estimation within Mplus (Muthén & Muthén, 1998–2007) along with four oblique rotation criteria to estimate the four factor structures (see Table 2). To solve the "alignment problem" for simulation studies, Mplus uses starting values provided by the user to ensure that factor order is consistent across replications (Asparouhov & Muthén, 2009).

Factor Structures

For the perfect simple structure (or *perfect cluster configuration*) condition, the first 15 variables/items ($i1$–$i15$) measured only Factor 1, whereas the last 15 variables/items ($i16$–$i30$) measured only Factor 2. For this factor structure, all factor pattern loadings were Large in magnitude on the dominate factors and ranged from .40 to .82 in increments of .03. Notice, there were no cross-loadings. The approximate simple structure test was identical to the perfect simple structure test, with the exception that the cross-loadings were not zero (i.e., small cross-loadings existed). The Small cross-loadings ranged from .02

TABLE 2
Simulated Factor Pattern Loadings for Each Factor Structure Condition

Item	Perfect Simple Structure		Approximate Simple Structure		Complex Structure		General Structure	
	λ_{i1}	λ_{i2}	λ_{i1}	λ_{i2}	λ_{i1}	λ_{i2}	λ_{i1}	λ_{i2}
1	**.76**	0	**.76**	.22	**.76**	.15	**.76**	.22
2	**.70**	0	**.70**	.26	**.70**	.26	**.70**	.26
3	**.58**	0	**.58**	.20	**.58**	.40	**.58**	.20
4	**.82**	0	**.82**	.08	**.82**	.20	**.82**	.12
5	**.46**	0	**.46**	.24	**.46**	.42	**.46**	.08
6	**.49**	0	**.49**	.12	**.49**	.24	**.49**	.24
7	**.79**	0	**.79**	.10	**.79**	.22	**.79**	.10
8	**.52**	0	**.52**	.04	**.52**	.46	**.52**	.04
9	**.43**	0	**.43**	.28	**.43**	.48	**.43**	.28
10	**.73**	0	**.73**	.14	**.73**	.28	**.73**	.14
11	**.64**	0	**.64**	.16	**.64**	.38	**.64**	.16
12	**.61**	0	**.61**	.06	**.61**	.34	**.61**	.06
13	**.40**	0	**.40**	.18	**.40**	.44	**.40**	.18
14	**.67**	0	**.67**	.02	**.67**	.32	**.67**	.02
15	**.55**	0	**.55**	.03	**.55**	.30	**.55**	.03
16	0	**.43**	.18	**.43**	.24	**.43**	**.43**	.18
17	0	**.82**	.03	**.82**	.20	**.82**	**.82**	.03
18	0	**.67**	.14	**.67**	.30	**.67**	**.67**	.14
19	0	**.70**	.16	**.70**	.22	**.70**	**.70**	.16
20	0	**.73**	.06	**.73**	.32	**.73**	**.73**	.06
21	0	**.40**	.22	**.40**	.38	**.40**	**.40**	.22
22	0	**.49**	.24	**.49**	.42	**.49**	**.49**	.24
23	0	**.58**	.10	**.58**	.48	**.58**	**.58**	.10
24	0	**.55**	.12	**.55**	.46	**.55**	**.55**	.12
25	0	**.64**	.28	**.64**	.34	**.64**	**.64**	.28
26	0	**.61**	.26	**.61**	.40	**.61**	**.61**	.24
27	0	**.76**	.04	**.76**	.28	**.76**	**.76**	.04
28	0	**.52**	.08	**.52**	.36	**.52**	**.52**	.08
29	0	**.79**	.02	**.79**	.26	**.79**	**.79**	.02
30	0	**.46**	.20	**.46**	.44	**.46**	**.46**	.20

Note. Factor pattern loadings in bold represent the Large factor pattern loadings, whereas the other factor pattern loadings represent the Zero, Small, and Medium factor pattern loadings.

to .28 in increments of .02. The complex structure test was also similar to the perfect simple structure test with the exception that moderate cross-loadings were used ranging from .20 to .48 in increments of .02. The general factor, which conceptually only has one factor with small factor pattern loadings on a second factor, was created to represent a factor structure commonly encountered in practice. For example, a researcher using a relatively new measure

might expect a single-factor solution but still test whether a second factor is present.

Correlation Between Factors and Rotation Criteria

To assess the impact of the interfactor correlation, eight interfactor population correlations (ρ) were employed: .00, .10, .20, .30, .40, .50, .60, and .70. The interfactor correlation should be considered when selecting a rotation criterion given that the traditional factor model assumes orthogonal factors, whereas the oblique rotation relaxes this assumption. Gorsuch (1983) indicated researchers can relax this assumption of orthogonal factors using oblique rotations assuming these correlations are relatively small. However, this statement's validity has not been explored and may vary depending on the rotation criterion employed.

Because most factors within the social sciences are correlated and orthogonal and oblique solutions will produce nearly identical results for uncorrelated factors (Harman, 1976), only oblique rotations were employed for this study. A large range of ρs were selected as it is unknown at what correlation magnitude each rotation performs well, as each rotation criterion has a different intended function (see introduction).

Rotation Criteria

As indicated in the Mplus manual, users may select from numerous oblique and orthogonal rotation criteria. Of these rotations, only the oblique criteria of Quartimin, CF-Equamax, CF-Facparsim, and Geomin were evaluated within this study for several reasons. First, only oblique rotations were explored as orthogonal rotations (e.g., Varimax, orthogonal CF- Equamax, etc.) are not theoretically appropriate with correlated factors (Fabrigar et al., 1999; Harman, 1976). Second, the Direct Quartimin is mathematically equivalent to the default CF-Quartimax, Crawfer, and Oblimin rotations (see Muthén & Muthén, 1998–2007, p. 487), as all values of k are set at zero, and therefore were not studied. Also, k was not varied as an infinite number of values could be specified. However, a researcher could change the default value of k to determine its influence on the factor pattern matrix or evaluate each rotation's equation. Third, given that $p = 30$ and $m = 2$ for this study, the CF-Equamax ($k = m/2p \approx .03$), CF-Varimax ($k = 1/p \approx .03$), and CF-Parsimax ($k = (m-1)/(p+m-2) \approx .03$) rotations will always yield identical results given that the k values were equal. Fourth, rotation criteria that performed poorly with oblique factor structures (i.e., McCammon's (1966) Minimum Entropy) and general factor structures (i.e., McKeon's (1968) Infomax) were not considered. Also not evaluated were those rotation criteria similar to CFA that require researchers to specify factor pattern matrix elements, such as the Target rotation criteria (i.e., Browne, 2001). Finally,

following the procedures of Browne (2001) and Asparouhov and Muthén (in press), only single-stage rotation criteria were evaluated and therefore the two-stage Promax rotation was not of interest.

Given that many of the rotations allow the users to specify k, a brief demonstration was also provided (see *Crawfer demonstration*) to illustrate the effect of varying k on the estimated factor pattern loadings and interfactor correlation. As seen from the aforementioned rotation criteria's description, users can determine the degree to which each rotation seeks to minimize either row or column complexity. Evaluating the Crawfer rotation equation, it is apparent that as k increases the amount of row complexity will increase, although this increase may not be linear.

Dependent Variables

For each simulation condition, the average $\hat{\lambda}_{imr}$, $\overline{\lambda}_{imr} = \frac{1}{R}\sum_{r=1}^{R}\hat{\lambda}_{imr}$, bias, $\lambda_{bias} = \hat{\lambda}_{imr} - \lambda_{im}$, and sampling error of $\hat{\lambda}_{im}$, $\overline{SE}_{emp_{im}} = \sqrt{\sum_{r=1}^{R}(\hat{\lambda}_{imr} - \overline{\lambda}_{imr})^2/(R-1)}$, results were obtained across the R replications. The factor pattern loading bias and sampling error results were reported in the following order: perfect simple structure, approximate simple structure, complex structure, and a general factor. For each factor structure the Large, Medium, Small, and Zero factor pattern loadings were evaluated independently. Separate analyses for each factor pattern loading magnitude ensured that the effects from one factor magnitude did not confound the interpretation of the other factor magnitudes. Finally, population rotation bias (λ_{PRB}) is defined as the differences between the rotated population data and the true population values for each of the four factor structures.[1] It is also worth noting that the same true population values were used for each rotation evaluated.

To ease the description of the results, bias was categorized into three groups: small ($|\lambda_{bias}| \leq .05$), medium ($.05 < |\lambda_{bias}| < .15$), and large ($|\lambda_{bias}| \geq .15$). These same guidelines were applied to ρ. To conserve space, the average bias and sampling error results were reported for each condition, with the individual factor pattern loadings bias and sampling error results available from the corresponding author. Instead, to assess the individual impact on the individual factor pattern loadings, a demonstration using the Crawfer rotation was presented that also showed the impact of varying k.

---

[1]Because EFA indeterminacy results in an infinite number of population factor pattern loading matrices and population factor correlation matrices, "bias" and "error" are not true indicators of deviance from truth. Bias and error could be more accurately described as "population differences." Due to convention, and because the results do reflect differences between some known population (generated matrix), we continue to use the terms "bias" and "error" throughout this article.

RESULTS

Perfect Simple Structure

Results in Table 3 revealed mixed findings depending on the rotation criterion employed and ρ values. The sampling error was consistently small across rotation criteria and conditions. In addition, the differences between the population rotation bias (λ_{PRB}) and average estimated bias (λ_{bias}) was largely negligible, which suggests that any differences in results are due to the experimental conditions. In light of this, population rotation bias and average estimated bias are discussed together as bias.

The Zero factor pattern loading condition often displayed slightly greater bias than the Large factor pattern loading condition, and differences were more pronounced based on the intended rotation criterion. Across ρ values for both Zero and Large factor pattern loading conditions, Geomin and Quartimin bias was always near zero ($\leq |.05|$). Conversely, both CF-Equamax and CF-Facparsim tended to slightly underestimate the Large factor pattern loadings while overestimating the Zero factor pattern loadings. Notice the amount of bias increased with the interfactor correlation (see Table 3).

In terms of the estimated interfactor correlations, the Geomin and Quartimin displayed unbiased results, whereas the CF-Equamax and CF-Facparsim tended to underestimate the interfactor correlation as ρ increased. CF-Facparsim consistently underestimated ρ the most, followed by CF-Equamax rotation.

Approximate Simple Structure

Similar to the perfect simple structure findings, the sampling error was generally small and consistent across rotation criteria and conditions with negligible differences between λ_{PRB} and λ_{bias} (Table 4). The one exception was Quartimin at $\rho = .70$, which had greater sampling error and λ_{PRB}. Thus, as before, λ_{PRB} and λ_{bias} are discussed together as bias. The results for the average estimated factor pattern loading conditions revealed small bias for the Large factor pattern loadings across rotation criteria, whereas bias was small ($|\lambda_{bias}| \leq .05$) to medium ($.05 < |\lambda_{bias}| < .15$) for the Small factor pattern loadings.

Unlike the perfect simple structure results, where bias was least pronounced for Geomin and Quartimin, the opposite was true for the approximate simple structure condition. Bias was most pronounced under the Small factor pattern condition for the Geomin and Quartimin rotations, which consistently underestimated the "true" factor pattern loadings. Consequently, these two rotations gave a closer impression of perfect simple structure as the average bias was nearly equal to the average Small true pattern loadings (i.e., $\overline{\lambda}_{im} = .14$). Conversely, CF-Equamax and CF-Facparsim tended to have minimal factor pattern loading bias,

88

TABLE 3
Simple Structure Population and Sample Results

Rotation Criterion	ρ							
	.00	.10	.20	.30	.40	.50	.60	.70
Large factor pattern loading condition ($\bar{\lambda}_{im} = .61$)								
CF-Equamax	**.00** .00 (.05)	**.00** .00 (.05)	**.00** .00 (.05)	**-.01** -.01 (.05)	**-.01** -.01 (.05)	**-.02** -.02 (.06)	**-.03** -.03 (.06)	**-.04** -.05 (.06)
CF-Facparsim	**.00** .00 (.05)	**.00** .00 (.05)	**.00** -.01 (.05)	**-.01** -.01 (.05)	**-.02** -.02 (.05)	**-.03** -.03 (.05)	**-.04** -.04 (.05)	**-.06** -.06 (.06)
Geomin	**.00** .00 (.05)	**.00** .00 (.05)	**.00** .00 (.07)	**.00** .00 (.06)	**.00** .00 (.06)	**.00** .00 (.06)	**.00** -.01 (.07)	**.00** -.01 (.07)
Quartimin	**.00** .00 (.07)	**.00** .00 (.07)	**.00** .00 (.07)	**.00** .00 (.07)	**.00** -.01 (.07)	**.00** -.01 (.07)	**.00** -.01 (.08)	**.00** -.01 (.07)
Zero factor pattern loading condition ($\bar{\lambda}_{im} = .00$)								
CF-Equamax	**.00** .00 (.05)	**.01** .01 (.05)	**.02** .02 (.05)	**.03** .03 (.05)	**.04** .04 (.05)	**.05** .05 (.06)	**.06** .05 (.05)	**.07** .08 (.06)
CF-Facparsim	**.00** .00 (.05)	**.02** .02 (.06)	**.03** .03 (.06)	**.05** .05 (.06)	**.06** .06 (.05)	**.08** .08 (.05)	**.10** .10 (.05)	**.12** .12 (.06)
Geomin	**.00** .00 (.05)	**.00** .00 (.05)	**.00** .00 (.06)	**.00** .00 (.05)	**.00** .01 (.05)	**.00** .01 (.05)	**.00** .01 (.06)	**.00** .02 (.07)
Quartimin	**.00** .00 (.06)	**.00** .00 (.06)	**.00** .00 (.06)	**.00** .01 (.06)	**.00** .01 (.06)	**.00** .02 (.07)	**.00** .02 (.08)	**.00** .02 (.07)
Interfactor correlations								
CF-Equamax	**.00** .00 (.04)	**-.03** -.03 (.04)	**-.06** -.06 (.04)	**-.09** -.09 (.04)	**-.11** -.12 (.04)	**-.14** -.14 (.04)	**-.15** -.16 (.04)	**-.16** -.18 (.03)
CF-Facparsim	**.00** .00 (.03)	**-.05** -.05 (.03)	**-.10** -.10 (.03)	**-.14** -.14 (.03)	**-.19** -.19 (.03)	**-.23** -.23 (.03)	**-.26** -.27 (.03)	**-.28** -.29 (.03)
Geomin	**.00** .00 (.06)	**.00** .00 (.06)	**.00** .00 (.06)	**.00** -.01 (.06)	**.00** -.02 (.05)	**.00** -.02 (.05)	**.00** -.02 (.04)	**.00** -.03 (.04)
Quartimin	**.00** .00 (.06)	**.00** -.01 (.06)	**.00** -.01 (.06)	**.00** -.02 (.06)	**.00** -.03 (.05)	**.00** -.04 (.05)	**.00** -.05 (.04)	**.00** -.03 (.03)

Note. $\bar{\lambda}_{im}$ represents the average true factor pattern loading within each subgroup. Population rotation bias values (bolded), average estimated factor pattern loading bias (located below the population values), and standard errors (in parentheses) for each factor pattern loading magnitude, along with the average estimated interfactor correlation bias and standard errors (in parentheses), under each simple structure condition.

TABLE 4
Approximate Simple Structure Population and Sample Results

Rotation Criterion	ρ							
	.00	.10	.20	.30	.40	.50	.60	.70
Large factor pattern loading condition ($\overline{\lambda}_{im} = .61$)								
CF-Equamax	**.00** -.01 (.05)	**.00** .00 (.05)	**.01** .01 (.05)	**.01** .01 (.06)	**.01** .01 (.06)	**.01** .00 (.06)	**.00** -.01 (.06)	**.00** -.01 (.05)
CF-Facparsim	**.00** -.01 (.05)	**.00** .00 (.05)	**.00** .00 (.05)	**.00** .00 (.05)	**.00** .00 (.05)	**.00** -.01 (.05)	**-.01** -.01 (.05)	**-.02** -.03 (.06)
Geomin	**.00** -.01 (.05)	**.01** .00 (.05)	**.01** .01 (.06)	**.02** .02 (.06)	**.03** .02 (.06)	**.03** .03 (.07)	**.03** .04 (.07)	**.04** .04 (.08)
Quartimin	**.00** .00 (.06)	**.01** .01 (.06)	**.02** .01 (.06)	**.03** .02 (.06)	**.03** .03 (.06)	**.04** .03 (.07)	**.05** .04 (.07)	**.05** .02 (.12)
Small factor pattern loading condition ($\overline{\lambda}_{im} = .14$)								
CF-Equamax	**-.08** -.08 (.05)	**-.07** -.06 (.05)	**-.06** -.04 (.05)	**-.05** -.03 (.05)	**-.03** -.02 (.05)	**-.02** -.01 (.05)	**-.01** .01 (.06)	**.00** -.08 (.06)
CF-Facparsim	**-.06** -.06 (.05)	**-.05** -.05 (.05)	**-.03** -.03 (.05)	**-.02** -.02 (.05)	**.00** .00 (.05)	**.02** .02 (.05)	**.03** .04 (.05)	**.05** .06 (.05)
Geomin	**-.09** -.09 (.05)	**-.09** -.08 (.05)	**-.08** -.08 (.05)	**-.08** -.08 (.05)	**-.07** -.07 (.06)	**-.07** -.07 (.06)	**-.06** -.07 (.07)	**-.06** -.06 (.08)
Quartimin	**-.10** -.09 (.05)	**-.09** -.09 (.05)	**-.09** -.09 (.05)	**-.09** -.08 (.05)	**-.09** -.09 (.06)	**-.08** -.08 (.06)	**-.08** -.08 (.07)	**-.08** -.05 (.13)
Interfactor correlations								
CF-Equamax	**.25** .25 (.04)	**.22** .21 (.04)	**.18** .17 (.04)	**.14** .13 (.03)	**.10** .09 (.03)	**.06** .05 (.03)	**.03** .01 (.03)	**-.01** -.02 (.02)
CF-Facparsim	**.21** .21 (.03)	**.16** .16 (.03)	**.11** .10 (.03)	**.06** .05 (.03)	**.01** .00 (.03)	**-.04** -.05 (.03)	**-.09** -.09 (.02)	**-.13** -.13 (.02)
Geomin	**.30** .29 (.06)	**.28** .27 (.05)	**.25** .25 (.05)	**.23** .22 (.05)	**.19** .19 (.04)	**.15** .16 (.04)	**.12** .13 (.03)	**.10** .10 (.03)
Quartimin	**.31** .30 (.05)	**.30** .28 (.05)	**.27** .26 (.04)	**.25** .23 (.04)	**.22** .20 (.04)	**.19** .17 (.03)	**.15** .14 (.03)	**.12** .05 (.18)

Note. $\overline{\lambda}_{im}$ represents the average true factor pattern loading within each subgroup. Population rotation bias values (bolded), average estimated factor pattern loading bias (located below the population values), and standard errors (in parentheses) for each factor pattern loading magnitude, along with the average estimated interfactor correlation bias and standard errors (in parentheses), under each approximate simple structure condition.

89

which is expected given that they both allow variances to be spread more equally across factors. In other words, they allow for a more complex factor structure and do not seek to impose a perfect simple structure.

The factor pattern loading results linked directly to the interfactor correlations. For every value of ρ, estimation bias was larger for the Geomin and Quartimin rotations. In fact, bias was as large as .31 for Quartimin when $\rho = 0$. Conversely, CF-Equamax and CF-Facparsim tended to produce less biased estimates of ρ, although these values were still medium to large in many cases. Again, notice the direct relationship between interfactor correlation and factor pattern loading bias.

These results are interesting from an applied standpoint because researchers may feel confident that large factor pattern loadings on the proper factor are relatively unbiased regardless of the rotation criterion. However, the cross-loading interpretations are dependent upon the selected rotation criterion. The Geomin and Quartimin rotations will provide cross-loadings closer to zero to depict perfect simple structure at the cost of increasing interfactor correlations. Conversely, the CF-Equamax and CF-Facparsim rotations will better estimate an approximate simple structure solution with less interfactor correlation bias.

It is clear that researchers must decide between minimizing the cross-loadings and reducing the interfactor correlations. From a statistical and measurement perspective, researchers often seek both properties and thus need to carefully consider their choice of rotation criterion.

Complex Structure

The complex structure results (see Table 5) partially mirror the approximate simple structure results, with the Moderate factor pattern loading condition having greater bias than the Large factor pattern loading condition. The one exception was Quartimin when $\rho \geq .40$, where the Large factor pattern loadings had greater bias. Focusing on the Large factor pattern loading conditions, CF-Equamax and CF-Facparsim consistently had small factor pattern bias (i.e., λ_{PRB} and λ_{bias}). For Geomin and Quartimin, bias increased drastically at $\rho = .60$ and $\rho = .40$, respectively, whereas sampling error was considerably larger between $\rho = .50$ and $\rho = .60$ for Geomin and $\rho = .30$ and $\rho = .50$ for Quartimin. The increased sampling error for these rotations was a direct result of factor collapse as samples commonly displayed a single-factor solution. However, sampling error for CF-Equamax and CF-Facparsim was generally small across the interfactor correlation conditions.

The Moderate factor pattern loading conditions produced different results depending on the interaction between the rotation criterion and ρ. Bias was always medium to large for the Moderate factor loading condition as each rotation criterion sought to decrease factor and/or variable complexity. Contrasting results oc-

TABLE 5
Complex Structure Population and Sample Results

Rotation Criterion	ρ							
	.00	.10	.20	.30	.40	.50	.60	.70
	Large factor pattern loading condition ($\overline{\lambda}_{im}$ = .61)							
CF-Equamax	**−.02**	**−.01**	**.00**	**.01**	**.02**	**.03**	**.03**	**.03**
	−.03 (.06)	−.01 (.06)	.00 (.06)	.01 (.06)	.01 (.06)	.02 (.06)	.03 (.06)	.03 (.06)
CF-Facparsim	**−.03**	**−.02**	**−.01**	**.00**	**.00**	**.01**	**.01**	**.01**
	−.03 (.05)	−.02 (.05)	−.01 (.05)	−.01 (.05)	.00 (.05)	.00 (.05)	.00 (.05)	.00 (.05)
Geomin	**−.01**	**.01**	**.02**	**.03**	**.05**	**.06**	**−.25**	**−.23**
	−.02 (.07)	.00 (.07)	.02 (.07)	.03 (.07)	.04 (.10)	.00 (.21)	−.14 (.25)	−.21 (.09)
Quartimin	**−.01**	**.00**	**.02**	**.04**	**−.29**	**−.27**	**−.25**	**−.23**
	−.02 (.06)	.00 (.07)	.02 (.07)	.00 (.16)	−.14 (.26)	−.25 (.12)	−.25 (.05)	−.23 (.04)
	Moderate factor pattern loading condition ($\overline{\lambda}_{im}$ = .33)							
CF-Equamax	**−.16**	**−.15**	**−.14**	**−.12**	**−.11**	**−.10**	**−.08**	**−.07**
	−.16 (.05)	−.15 (.05)	−.13 (.05)	−.12 (.05)	−.11 (.05)	−.10 (.05)	−.08 (.05)	−.07 (.05)
CF-Facparsim	**−.14**	**−.12**	**−.10**	**−.09**	**−.07**	**−.05**	**−.04**	**−.02**
	−.13 (.05)	−.12 (.05)	−.10 (.05)	−.09 (.05)	−.07 (.05)	−.05 (.05)	−.03 (.04)	−.02 (.04)
Geomin	**−.20**	**−.19**	**−.17**	**−.16**	**−.16**	**−.15**	**.17**	**.17**
	−.19 (.06)	−.18 (.06)	−.17 (.07)	−.17 (.07)	−.16 (.10)	−.10 (.20)	.06 (.24)	.15 (.10)
Quartimin	**−.19**	**−.19**	**−.18**	**−.17**	**.17**	**.16**	**.16**	**.16**
	−.19 (.06)	−.18 (.06)	−.18 (.06)	−.14 (.15)	.01 (.25)	.14 (.12)	.16 (.05)	.16 (.04)
	Interfactor correlations							
CF-Equamax	**.55**	**.50**	**.44**	**.38**	**.32**	**.26**	**.20**	**.14**
	.54 (.03)	.49 (.03)	.43 (.03)	.37 (.02)	.31 (.02)	.25 (.02)	.19 (.02)	.13 (.01)
CF-Facparsim	**.48**	**.42**	**.35**	**.29**	**.22**	**.16**	**.10**	**.04**
	.47 (.03)	.41 (.02)	.35 (.02)	.28 (.02)	.22 (.02)	.16 (.02)	.09 (.02)	.04 (.02)
Geomin	**.64**	**.59**	**.51**	**.46**	**.40**	**.34**	**−.53**	**−.65**
	.62 (.04)	.57 (.04)	.52 (.03)	.46 (.03)	.39 (.10)	.20 (.30)	−.28 (.38)	−.64 (.10)
Quartimin	**.63**	**.58**	**.53**	**.47**	**−.27**	**−.43**	**−.56**	**−.67**
	.62 (.04)	.57 (.03)	.52 (.04)	.40 (.19)	.00 (.35)	−.39 (.15)	−.55 (.03)	−.67 (.02)

Note. $\overline{\lambda}_{im}$ represents the average true factor pattern loading within each subgroup. Population rotation bias values (bolded), average estimated factor pattern loading bias (located below the population values), and standard errors (in parentheses) for each factor pattern loading magnitude, along with the average estimated interfactor correlation bias and standard errors (in parentheses), under each complex structure condition.

curred between rotation criteria because CF-Equamax and CF-Facparsim sought to reduce factor complexity, whereas Geomin and Quartimin attempted to reduce row complexity.

The Geomin and Quartimin rotation criteria with larger interfactor correlations often resulted in larger sampling error. This was true of both the Large and Moderate factor pattern loading conditions due to factor collapse in some samples. Stated differently, factor collapse produced larger sampling error because all large factor pattern loadings (i.e., $\hat{\lambda}_{i1} > .70$) loaded on Factor 1 and all small factor pattern loadings (i.e., $\hat{\lambda}_{i1} < .30$) loaded on Factor 2 for some samples. However, the increase in sampling error was only temporary for the Quartimin criterion as eventually only single-factor solutions emerged at $\rho = 60$.

Ignoring the aforementioned conditions that frequently exhibited factor collapse, the interfactor correlations were always very biased due to reduced cross-loading magnitudes. This finding is critical to remember as variables that measure more than one factor will always increase the interfactor correlation in order to minimize the cross-loadings.

In summary, only the Large factor pattern loading estimates can be interpreted with any degree of confidence for the CF-Equamax, CF-Facparsim, and Geomin rotation criteria. Conversely, the Quartimin rotation criterion only creates the two-factor solution with very small interfactor correlations as factor collapse often occurred at $\rho \geq .40$. As data are rotated to minimize row or column complexity, the cross-loadings are decreased resulting in larger interfactor correlations. The complex factor structure presented an interesting scenario for researchers as results can vary significantly depending on the rotation criterion utilized.

General Structure

The general factor structure produced significantly different results depending on the rotation criterion utilized (see Table 6). The CF-Equamax and CF-Facparsim rotations, on average, always underestimated the Large factor pattern loadings (i.e., Factor 1) and overestimated the Small factor pattern loadings (i.e., Factor 2). Under these conditions, the average bias results were very misleading as these rotations attempted to minimize factor complexity. Consequently, these rotations produced very large and small estimated factor pattern loadings within Factors 1 and 2. Conversely, the Geomin and Quartimin rotations estimated the Large factor pattern loadings (Factor 1) well, although estimation error was slightly larger for Factor 2. This is because Geomin and Quartimin rotations attempt to minimize row complexity; thus the average estimated factor pattern loadings were close to zero (i.e., $\overline{\lambda}_{im} + \overline{\hat{\lambda}}_{im} = 0$) on Factor 2. Unlike the CF-Equamax and CF-Facparsim rotations, these rotations give the impression of a single factor with near perfect simple structure.

TABLE 6
General Structure Population and Sample Results

Rotation Criterion	ρ							
	.00	.10	.20	.30	.40	.50	.60	.70
	Large factor one pattern loading condition ($\bar{\lambda}_{im}$ = .61)							
CF-Equamax	**−.28**	**−.27**	**−.27**	**−.26**	**−.26**	**−.25**	**−.25**	**NC**
	−.27 (.17)	−.27 (.17)	−.26 (.17)	−.26 (.17)	−.25 (.17)	−.24 (.17)	−.24 (.17)	−.23 (.17)
CF-Facparsim	**−.27**	**−.26**	**−.26**	**−.25**	**−.25**	**−.24**	**−.24**	**NC**
	−.27 (.14)	−.26 (.14)	−.26 (.14)	−.25 (.14)	−.24 (.14)	−.24 (.15)	−.23 (.15)	−.23 (.15)
Geomin	**.02**	**.03**	**.04**	**.06**	**.07**	**.08**	**.09**	**NC**
	.00 (.10)	.02 (.09)	.03 (.09)	.04 (.09)	.06 (.08)	.08 (.05)	.09 (.05)	.10 (.09)
Quartimin	**.02**	**.03**	**.04**	**.06**	**.07**	**.08**	**.09**	**NC**
	.00 (.10)	.01 (.10)	.03 (.09)	.04 (.09)	.06 (.09)	.07 (.07)	.08 (.09)	.09 (.11)
	Small factor two pattern loading condition ($\bar{\lambda}_{im}$ = .14)							
CF-Equamax	**.18**	**.19**	**.19**	**.20**	**.20**	**.21**	**.21**	**NC**
	.19 (.17)	.19 (.17)	.20 (.17)	.21 (.17)	.21 (.17)	.22 (.17)	.22 (.17)	.23 (.17)
CF-Facparsim	**.18**	**.19**	**.20**	**.21**	**.21**	**.22**	**.22**	**NC**
	.19 (.14)	.20 (.14)	.21 (.14)	.22 (.14)	.22 (.14)	.23 (.15)	.24 (.15)	.24 (.15)
Geomin	**−.13**	**−.13**	**−.14**	**−.14**	**−.14**	**−.14**	**−.14**	**NC**
	−.11 (.14)	−.12 (.14)	−.12 (.14)	−.12 (.14)	−.12 (.13)	−.13 (.11)	−.13 (.11)	−.12 (.13)
Quartimin	**−.13**	**−.13**	**−.13**	**−.14**	**−.14**	**−.14**	**−.14**	**NC**
	−.11 (.14)	−.11 (.14)	−.12 (.14)	−.12 (.14)	−.12 (.14)	−.13 (.12)	−.12 (.13)	−.12 (.15)
	Interfactor correlations							
CF-Equamax	**.85**	**.76**	**.67**	**.58**	**.49**	**.40**	**.32**	**NC**
	.76 (.05)	.67 (.05)	.58 (.04)	.49 (.04)	.39 (.04)	.30 (.04)	.21 (.05)	.11 (.06)
CF-Facparsim	**.76**	**.67**	**.58**	**.49**	**.41**	**.32**	**.24**	**NC**
	.68 (.02)	.59 (.03)	.51 (.02)	.41 (.02)	.32 (.02)	.23 (.02)	.14 (.02)	.05 (.02)
Geomin	**.03**	**−.08**	**−.18**	**−.29**	**−.40**	**−.50**	**−.60**	**NC**
	.12 (.16)	.00 (.12)	−.10 (.13)	−.20 (.13)	−.31 (.12)	−.42 (.08)	−.52 (.09)	−.61 (.13)
Quartimin	**.04**	**−.07**	**−.18**	**−.29**	**−.40**	**−.50**	**−.61**	**NC**
	.13 (.17)	.02 (.17)	−.08 (.17)	−.18 (.17)	−.28 (.17)	−.37 (.17)	−.47 (.18)	−.56 (.19)
	Percent of samples that converged							
	38.2	34.2	29.7	26.5	23.6	20.6	18.0	16.1

Note. $\bar{\lambda}_{im}$ represents the average true factor pattern loading within each subgroup. Population conditions marked with NC did not converge. Population rotation bias values (bolded), average estimated factor pattern loading bias (located below the population values), and standard errors (in parentheses) for each factor pattern loading magnitude, along with the average estimated interfactor correlation bias and standard errors (in parentheses), under each general structure condition.

93

Finally, ρ values were always very biased, which again resulted from the intended rotation function. For this test condition, CF-Equamax and CF-Facparsim always overestimated ρ, whereas Geomin and Quartimin underestimated ρ. As seen in Table 6, one limitation of this condition was the number of samples that converged, with the rotated population rotation values also failing to converge when ρ = .70.

Crawfer Demonstration

For the Crawfer demonstration, rotated population results with approximate simple structure, complex structure, and a general factor when ρ = .40 were used to illustrate the impact of different values of *k* within the Crawfer rotation criterion. Perfect simple structure was not evaluated given that this condition would perform well regardless of rotation. Notice that the factor structures were identical to the larger simulation study to increase comparability.

Recall, the other rotation criteria compared in this study compute *k* based on the number of variables and factors. The Crawfer rotation criterion allows researchers to select values of *k* to specify whether row or column complexity is more desirable. This demonstration provides and discusses each individual factor pattern loading rather than discussing the group means. This was done to provide a more concrete example for readers and to focus on the complete factor pattern loading matrix. Within this example, Crawfer values for *k* were set at 0 (notice, this is equal to the Quartimin rotation) to increase row complexity and .10 to increase factor complexity. Values of *k* greater than .10 did not significantly alter the results.

Approximate Simple Structure

The approximate simple structure results in Table 7 show that the true factor pattern loadings (see Table 2) were more accurately estimated as *k* increased. When *k* = 0, the average Small factor pattern loadings were close to zero (*M* = .05) and all of the individual factor pattern loadings were underestimated and in many cases negative. Perhaps more important, the interfactor correlation was overestimated (*r* = .63) at the expense of reducing the cross-loadings. Based on these results, a researcher would likely conclude that most of the variables were relatively perfect measures of Factors 1 or 2 and the interfactor correlation was relatively high.

With a small change in *k* (*k* = .10), a researcher would likely still conclude that each variable reasonably represents each factor. However, concern may be warranted given that several of the estimated factor pattern loadings started to approach the common minimum threshold of .30, which suggests that the variable is partially measuring both factors. This change in rotation criterion

TABLE 7
Crawfer Population Rotated Results

Item	Approximate Simple Structure				Complex Structure				General Factor			
	Crawfer k = .00		Crawfer k = .10		Crawfer k = .00		Crawfer k = .10		Crawfer k = .00		Crawfer k = .10	
	$\hat{\lambda}_{i1}$	$\hat{\lambda}_{i2}$	$\hat{\lambda}_{i1}$	$\hat{\lambda}_{i2}$	$\hat{\lambda}_{i1}$	$\hat{\lambda}_{i2}$	$\hat{\lambda}_{i1}$	$\hat{\lambda}_{i2}$	$\hat{\lambda}_{i1}$	$\hat{\lambda}_{i2}$	$\hat{\lambda}_{i1}$	$\hat{\lambda}_{i2}$
1	**.81**	.09	**.77**	.19	**.95**	−.15	**.79**	.22	**.87**	.04	**.52**	.38
2	**.74**	.14	**.71**	.23	**.96**	−.09	**.70**	.33	**.83**	.09	**.58**	.29
3	**.62**	.10	**.59**	.18	**.66**	−.14	**.62**	.08	**.68**	.06	**.46**	.26
4	**.88**	.10	**.83**	.21	**1.03**	−.15	**.85**	.25	**.94**	.05	**.57**	.41
5	**.50**	−.01	**.47**	.06	**.74**	−.01	**.43**	.37	**.50**	−.02	**.22**	.29
6	**.53**	.03	**.49**	.10	**.62**	−.09	**.51**	.15	**.55**	.01	**.30**	.28
7	**.86**	−.05	**.80**	.07	**.86**	−.21	**.86**	.05	**.83**	−.06	**.31**	.55
8	**.57**	−.06	**.53**	.02	**.68**	−.09	**.53**	.19	**.53**	−.06	**.16**	.39
9	**.44**	.22	**.43**	.27	**.68**	−.01	**.40**	.33	**.58**	.15	**.57**	.04
10	**.79**	.01	**.74**	.11	**1.00**	−.10	**.73**	.35	**.80**	−.02	**.37**	.46
11	**.69**	.05	**.65**	.14	**.94**	−.05	**.62**	.39	**.72**	.01	**.40**	.35
12	**.67**	−.06	**.62**	.04	**.80**	−.10	**.62**	.24	**.63**	−.06	**.21**	.45
13	**.42**	.11	**.40**	.17	**.60**	−.02	**.38**	.27	**.50**	.07	**.39**	.13
14	**.74**	−.11	**.68**	−.01	**.79**	−.15	**.71**	.13	**.67**	−.11	**.15**	.54
15	**.61**	−.08	**.56**	.01	**.72**	−.09	**.56**	.21	**.56**	−.08	**.14**	.44
16	.14	**.43**	.17	**.43**	.55	.08	.17	**.44**	.52	.07	**.39**	.15
17	−.08	**.88**	.01	**.83**	.95	.20	.19	**.86**	.82	−.13	**.20**	.66
18	.06	**.70**	.13	**.67**	.80	.16	.18	**.70**	.74	.00	**.37**	.40
19	.08	**.73**	.15	**.70**	.75	.20	.08	**.75**	.78	.00	**.41**	.40
20	−.04	**.78**	.05	**.73**	.86	.17	.19	**.76**	.75	−.09	**.23**	.55
21	.19	**.39**	.22	**.40**	.65	.02	.34	**.37**	.52	.11	**.46**	.08
22	.20	**.48**	.23	**.48**	.64	.09	.20	**.50**	.62	.11	**.51**	.13
23	.03	**.61**	.09	**.58**	.88	.05	.41	**.55**	.63	−.03	**.28**	.38
24	.06	**.57**	.11	**.55**	.84	.04	.40	**.52**	.61	.00	**.31**	.32
25	.22	**.64**	.27	**.63**	.69	.19	.07	**.69**	.79	.11	**.61**	.21
26	−.04	**.65**	.03	**.61**	.84	.09	.31	**.60**	.62	−.08	**.18**	.47
27	.18	**.77**	.25	**.76**	.97	.15	.30	**.77**	.89	.07	**.59**	.34
28	.01	**.55**	.07	**.52**	.73	.07	.29	**.51**	.56	−.03	**.23**	.35
29	−.09	**.85**	.00	**.80**	.86	.22	.10	**.85**	.79	−.13	**.17**	.65
30	.16	**.46**	.19	**.46**	.75	.01	.40	**.42**	.56	.08	**.43**	.16
L	.65 (.61)		.61 (.61)		.46 (.61)		.62 (.61)		.68 (.61)		.36 (.61)	
S/M	.05 (.14)		.13 (.14)		.34 (.34)		.24 (.34)		.00 (.14)		.35 (.14)	
r	.63 (.40)		.45 (.40)		.05 (.40)		.70 (.40)		.00 (.40)		.84 (.40)	

Note. The true/simulated factor pattern loadings are in Table 2. Estimated factor pattern loadings in bold represent the Large factor pattern loadings. L represents the average Large estimated factor pattern loadings, S/M signifies the average Small or Moderate estimated factor pattern loadings, respectively, and r is the estimated interfactor correlation. Values in parentheses represent the average true or simulated values. Values of k were varied when $\rho = .40$ using the Crawfer rotation.

also produced smaller interfactor correlations ($r = .45$). Based on these results, researchers would likely conclude that several variables/items assess both factors with moderately large interfactor correlations, which slightly contrasts the conclusion when $k = 0$.

Complex Structure

As might be expected, the complex structure provided a more interesting example of the significance of selecting k and its influence on either the row or the column complexity function. Based solely on the factor pattern matrix, a researcher would likely conclude that a single factor exists when $k = 0$ as the cross-loadings were always less than .30 and in many cases close to zero. Based on this conclusion, the interfactor correlation ($r = .05$) would likely be of little interest. These results further explain the Quartimin finding in Table 5 when $\rho = .40$, where the average estimated factor pattern loadings were moderate in size for both the Large and Moderate condition because only a single factor emerged.

Similar to the approximate simple structure example, a small change in k produced a very different result. When $k = .10$, a researcher would likely conclude a two-factor solution based exclusively on the estimated factor pattern loadings. In addition, slight concerns may arise given that several items are measuring both factors and the interfactor correlation ($r = .70$) was high. Despite the high interfactor correlation concern, using a complexity function not focused on row complexity may help researchers purify a factor by removing those variables that cross-load. This may ultimately portray the factor structure in a different light. It is also important to remember that removing variables with larger cross-loadings should reduce the interfactor correlation. Overall, this complex structure provides an interesting example of how varying k influences the factor pattern matrix, thus leaving the researchers perplexed about how to interpret the factor solution.

General Factor Structure

Paralleling the other factor pattern structures, the interpretation of the general factor structure varied significantly depending on k. When $k = 0$, which equals the Quartimin rotation results when $\rho = .40$ in Table 6, the results in Table 7 suggest a single-factor solution with no interfactor correlation ($r = .00$) and very small cross-loadings. A slight change in k to .10 now presents the perception that each variable measures both factors. As indicated earlier, factor pattern loadings within each factor are being minimized and maximized to reduce the factor complexity. From the results when $k = .10$, a researcher might be perplexed about how to interpret the factor structure matrix given the large number of

cross-loadings and the large interfactor correlation ($r = .84$). These results demonstrate how a slight change in k can significantly impact the factor pattern structure interpretation, not to mention the interfactor correlations.

Taken collectively, the complex and general factor results display the importance of considering both the model fit (i.e., methods to ascertain the number of factors) and the rotation criterion when conducting an EFA as both are significant when interpreting the factor structure. Moreover, in both examples the interfactor correlation changed drastically as a result of k.

DISCUSSION

This study reviewed several rotation criteria and their capability to reproduce different factor pattern matrices. As Browne (2001) stated, there may be different circumstances when hypothesized factor structures call for a particular rotation criterion. The goal of the current study was to compare several rotation criteria in such situations, and begin to provide guidelines or insight for researchers using EFA. When selecting a rotation criterion for data lacking perfect simple structure, researchers must select between (a) estimating factor solutions with smaller cross-loadings and potentially larger interfactor correlations or (b) identifying more independent factors (i.e., smaller interfactor correlations) and slightly larger cross-loadings. We speculate that researchers will choose smaller cross-loadings and higher interfactor correlations with the notion that if the interfactor correlations are too large, either a factor would be dropped or, possibly, a second-order factor would be created. Nevertheless, the central emphasis on selecting the "most appropriate" rotation criterion should depend on the research question and the hypothesized factor structure. If researchers seek to uncover and eliminate variables that measure multiple factors, they may benefit from increasing the value of k or selecting a rotation criterion that does not seek to minimize row complexity, such as CF-Facparsim. Moreover, it is important for researchers to remember that removing variables with large cross-loadings ultimately should reduce the interfactor correlations, thus producing a solution with smaller cross-loadings and a smaller interfactor correlation.

Historically, researchers have selected a rotation criterion based on either its popularity or simply whether or not the factors are hypothesized to be correlated. As factor loading patterns can differ significantly based on the rotation criterion choice, this article argues that more attention is needed when selecting a rotation criterion rather than simply considering the orthogonality and obliqueness of the factors. Along with providing more justification for the rotation criterion selected, reporting multiple factor pattern matrices from different rotation criteria may also be necessary. This allows the readers to draw their own conclusions based on the competing factor structures.

It is instructive to consider the principles that guide different rotation criteria (see Table 1) when discussing the results. As anticipated, the rotation criteria investigated performed reasonably well for perfect simple structure. Each rotation was successful at maximizing or minimizing the pattern loadings factor in either the row or column depending on the intended rotation function. However, given that Geomin and Quartimin both place more weight on minimizing variable complexity, it is not surprising that they slightly outperformed CF-Equamax and CF-Facparsim, which seek to reduce factor complexity.

With more complex data structures, CF-Equamax and CF-Facparsim displayed less bias as more emphasis was placed on factor complexity as opposed to row complexity. Notice that CF-Facparsim performed slightly "better" than CF-Equamax, as CF-Equamax attempted to minimize the function by weighting both variable and factor complexity, whereas CF-Facparsim only minimizes factor complexity (see Table 1).

Though it is difficult to argue that any rotation is wrong statistically or mathematically, from a conceptual standpoint rotations that place less weight on row complexity offer more valuable information related to cross-loadings. Perhaps more important, CF-Equamax and CF-Facparsim did not experience factor collapse with larger cross-loadings and interfactor correlations as Quartimin and Geomin yielded unstable solutions under certain conditions. Notice that the Quartimin criterion experienced greater factor collapse than Geomin because it required a greater number of zero loadings (see Browne, 2001). Furthermore, these conclusions support past research indicating Quartimin (referred to as Quartimax in Crawford & Ferguson, 1970) tends to produce a general factor and Geomin functions poorly with more complex factor patterns loading matrices (Asparouhov & Muthén, 2009). On a positive note, these rotations tended to produce solutions that were easier to interpret for less complicated factor structures, as they focused on row complexity and reducing cross-loading magnitudes. Browne (2001) and Crawford (1975) noted that factor collapse is less probable under the CF criterion, which was also supported in this study.

In any case, it is critical to consider how the rotation criterion influences the factor pattern loadings. For example, as indicated previously, CF-Equamax, CF-Varimax, and CF-Parsimax would produce identical solutions as the value of k was equal. However, if the number of factors increased to three, but the number of variables remained at 30, CF-Varimax ($k \approx .03$) would place greater emphasis on row complexity than CF-Equamax ($k = .05$) and CF-Parsimax ($k \approx .07$). As seen in Table 1, researchers can easily calculate the value of k based on the number of factors and variables to determine the degree of variable and factor complexity.

It is both fortunate and unfortunate that there are numerous rotation criteria available to researchers. It is fortunate because researchers have access to multiple rotation criteria to facilitate the exploration of a variety of research questions

by helping to understand one's data. It is unfortunate because it can be arduous to select an appropriate rotation criterion or decide which rotation is "best" if conflicting solutions exist. For instance, looking at the Crawfer example with complex structure or a general factor, the researcher must decide whether to use $k = 0$ or $k = .10$ as they provide contradictory conclusions. Therefore, researchers can and should use other information available to them, such as model fit statistics and prior factor analytic research studies.

In reality, the selection of "best" rotation criterion must be made by the researcher. In many circumstances, different rotation criteria yield nearly identical rotated pattern loading matrices. When this does not happen, the researcher must make the difficult decision of choosing a rotation criterion that focuses more on row or column complexity or balances each with the understanding that this may have a significant impact on the cross-loading magnitudes and interfactor correlations.

Despite the emphasis on "estimation error or bias" as an assessment of rotation performance within this article, it is important to be cognizant of the limitations of EFA in rotating to a "correct" solution. As suggested by Thurstone (1947), the unrotated factor solution is rotated to acquire a factor pattern matrix that is easier to interpret, which consequently is often closer to perfect simple structure or approximate simple structure. Recall there is no single "correct" rotation solution due to the indeterminacy problem, as the model fit does not change based on the rotation selected (Mulaik, 2005). Instead, the rotation criterion simply redistributes the variance of each variable across the rotated factors to provide a more easily interpretable solution. Therefore, for a lack of better words, "estimation error" or "bias" was used throughout this article to assess an approximation of closeness to truth. It is vital to recognize that the term "bias" simply implies that the estimated values (i.e., factor pattern loadings and interfactor correlations) differed from the simulated values and not that these values are necessarily incorrect. As Asparouhov and Muthén (2009) point out, one must make the assumption that the rotated factor pattern loading matrix is the true rotated matrix so the simulation study is interpretable. Now of course, a researcher could choose another rotated factor pattern loading matrix as being the optimal rotated matrix; thus this alternative matrix would have to be considered. For a better understanding of the connection between indeterminacy and these results, the interested reader is referred to Mulaik (2005) and Asparouhov and Muthén (2009).

The purpose of this article is not to provide a definitive answer to which rotation criterion is "best," but to raise awareness of the fact that many different rotation criteria exist and that they may produce conflicting pattern loading matrices under certain conditions. Ultimately it is up to the researcher to fine-tune the rotation criterion (e.g., adjusting $\in$ in Geomin or k in the Crawfer) in an effort to make the "best" choice in rotation criterion.

EFA for Instrument Development and Validation

The factor pattern loadings within EFA provide essential information for researchers, especially with regard to test construction and validation. Even when researchers create items grounded in theory, EFA can be an important step in the initial instrument validation process. As this study demonstrated, the rotation criterion selected is central when conducting an EFA as it provides vital information connected to the factor structure complexity and the interfactor correlations. This has significant consequences for construct validity, dimensionality, and those items the researcher designates to measure each construct. For example, it is possible that some items correlate with multiple factors, which suggests these items should be revised or removed. However, if the rotated solution provides a factor pattern matrix with very small cross-loadings and a higher interfactor correlation, this information may be lost. Moreover, the researcher may conclude that the constructs have poor discriminate validity as items that measure multiple factors tend to increase the interfactor correlation. From a statistical standpoint, this solution is not incorrect. From a measurement perspective, however, essential information associated with each item's uniqueness may be concealed at the cost of artificially inflating the interfactor correlations. Finally, the rotation criterion selected may significantly influence theory development as EFA results often guide future research using those items or factors in later statistical analyses (e.g., structural equation models, multiple regression analyses, etc.).

Limitations

Despite the insight provided by this study, several important limitations should be noted. First, not all rotation criteria were evaluated and the same rotations may not perform similarly in other software packages. As pointed out by a reviewer, there are numerous rotation criteria and each has a different intended function. Consequently, this creates an epistemological problem for EFA because the rotation criterion chosen, which may be arbitrary, could influence the interpretation of the results. It is then critical for researchers to justify their choice of a rotation criterion and interpret the results while keeping in mind that the selected rotation significantly impacts the factor pattern matrix.

Like all simulation studies, this study has limited generalizability due to the conditions considered. It may be that other factor structures will result in different findings, especially with other rotation criteria. More specifically, the number of variables and factors investigated, along with the factor pattern loading magnitudes, may not generalize to solutions with more than two factors and a different number of variables/items. In a positive light, the work of Asparouhov and Muthén (2009), Browne (2001), and Crawford and Ferguson (1970) provides generalizability evidence associated with this study. These articles are also

excellent references for additional insight into how the number of variables and factors influence each rotation criterion. Users can also calculate how adding or removing variables and factors will influence row and column complex within the CF family (see Table 1). This study does touch on an important range of conditions researchers may experience, although additional research is needed to investigate conditions with greater diversity.

Another limitation is the type of data generated. In most cases, data are not multivariate normal with continuous variables. As indicated earlier, this type of data was selected to purify the design of confounding variables. Assuming the correct estimation method and correlation matrix is employed, it is believed that the results should not differ significantly given that each rotation criteria seeks to minimize a complexity function. Nevertheless, additional research is needed to confirm this notion with categorical data and other data structures. Moreover, it would be interesting to learn if variables with varying distributions influence the rotation criteria.

CONCLUSIONS

As demonstrated in this study, the selected rotation criterion can have profound effects on the estimated factor pattern loadings and interfactor correlations. This is of concern given that two researchers who analyze the same data may draw different conclusions simply because they used a different rotation criterion. Given the indeterminacy problem in EFA, "the choice of the best solution therefore cannot be made automatically and without human judgment" (Browne, 2001, p. 145). There is no right or wrong rotation criterion but instead the goal is to select the rotation that provides the simplest and most informative solution (Asparouhov & Muthén, 2009). Thus, researchers should consider the rotation best suited to address the research goal (Kass & Tinsley, 1979). It is important that researchers rely on strong theoretical justifications when choosing a rotation criterion as the type of factor structure hypothesized should determine the rotation criterion employed. Consequently, researchers should avoid traditional practice of selecting a rotation criterion, such as Varimax, simply because it provides an easily interpretable solution. Instead, it is vital for researchers to realize that their choice of a rotation criterion will significantly impact the manifestation of the hypothesized factor structure.

For these reasons, it is critical that researchers consider the rotation criterion selected and how it influences the interpretation of the results. The conclusions related to the number of factors may not differ but instead the degree of factor complexity or the decision related to whether or which variables should be removed. Researchers should also be cognizant when comparing factor analytic results across different studies as the conclusions can vary based on the selected

rotation criterion as well as numerous other reasons (i.e., sampling characteristics, estimation method or correlation matrix employed, etc.). Overall, this study demonstrated the significance of selecting a rotation criterion and the implications when selecting a rotation that seeks to acquire row or column complexity.

REFERENCES

Asparouhov, T., & Muthén, B. (2009). Exploratory structural equation modeling. *Structural Equation Modeling, 16,* 397–438.

Browne, M. W. (2001). An overview of analytic rotation in exploratory factor analysis. *Multivariate Behavioral Research, 36,* 111–150.

Carroll, J. B. (1953). An analytic solution for approximating simple structure in factor analysis. *Psychometrika, 18,* 23–38.

Crawford. C. B. (1975). Determining the number of interpretable factors. *Psychological Bulletin, 82,* 226–237.

Crawford, C. B., & Ferguson, G. A. (1970). A general rotation criterion and its use in orthogonal rotation. *Psychometrika, 35,* 321–332.

Comrey, A. L., & Lee, H. B. (1992). *A first course in factor analysis.* Hillsdale, NJ: Erlbaum.

de Vet, H. C. W., Adèr, H. J., Terwee, C. B., & Pouwer, F. (2005). Are factor analytical techniques used appropriately in the validation of status questionnaires? A systematic review on the quality of factor analysis of the SF-36. *Quality of Life Research, 14,* 1203–1218.

Fabrigar, L. R., Wegener, D. T., MacCallum, R. C., & Strahan, E. J. (1999). Evaluating the use of exploratory factor analysis in psychological research. *Psychological Methods, 4,* 272–299.

Finch, H. (2006). Comparison of the performance of Varimax and Promax rotations: Factor structure recovery for dichotomous items. *Journal of Educational Measurement, 43*(1), 39–52.

Flora, D. B., & Curran P. J. (2004). An empirical evaluation of alternative methods of estimation for confirmatory factor analysis with ordinal data. *Psychological Methods, 9,* 466–491.

Ford, J. K., MacCallum, R. C., & Tait, M. (1986). The application of exploratory factor analysis in applied psychology: A critical review and analysis. *Personnel Psychology, 39,* 291–314.

Gorsuch, R. L. (1983). *Factor analysis* (2nd ed.). Hillsdale, NJ: Erlbaum.

Harman, H. H. (1976). *Modern factor analysis* (3rd ed.). Chicago: The University of Chicago Press.

Hayton, J. C., Allen, D. G., & Scarpello, V. (2004). Factor retention decisions in exploratory factor analysis: A tutorial on parallel analysis. *Organizational Research Methods, 7,* 191–205.

Henson, R. K., & Roberts, J. K. (2006). Use of exploratory factor analysis in published research: Common errors and some comment on improved practice. *Educational and Psychological Measurement, 66,* 393–416.

Hogarty, K. Y., Hines, C. V., Kromrey, J. D., Ferron, J. M., & Mumford, K. R. (2005). The quality of factor solutions in exploratory factor analysis: The influence of sample size, communality, and overdetermination. *Educational and Psychological Measurement, 65,* 202–226.

Hurley, A. E., Scandura, T. A., Schriesheim, C. A., Brannick, M. T., Seers, A., Vandenberg, R. J., et al. (1997). Exploratory and confirmatory factor analysis: Guidelines, issues, and alternatives. *Journal of Organizational Behavior, 18,* 667–683.

Jennrich, R. I. (2007). Rotation methods, algorithms, and standard errors. In R. Cudeck & R. C. MacCallum (Eds.), *Factor analysis at 100: Historical developments and future directions* (1st ed., pp. 315–335). Mahwah, NJ: Erlbaum.

Jöreskog, K., & Moustaki, I. (2001). Factor analysis for ordinal variables: A comparison of three approaches. *Multivariate Behavioral Research, 36,* 347–387.

Kaiser, H. F. (1958). The varimax criterion for analytic rotation in factor analysis. *Psychometrika, 23,* 187–200.

Kass, R. A., & Tinsley, H. E. A. (1979). Factor analysis. *Journal of Leisure Research, 11,* 120–138.

MacCallum, R. C., Roznowski, M., & Necowitz, L. B. (1992). Model modification in covariance structure analysis: The problem of capitalization on chance. *Psychological Bulletin, 111,* 490–504.

MacCallum, R. C., Widaman, K. F., Zhang, S., & Hong, S. (1999). Sample size in factor analysis. *Psychological Methods, 4,* 84–99.

McCammon, R. B. (1966). Principal component analysis and its application in large-scale correlation studies. *Journal of Geology, 74,* 721–733.

McDonald, R. P. (2005). Semiconfirmatory factor analysis: The example of anxiety and depression. *Structural Equation Modeling, 12,* 163–172.

McKeon, J. J. (1968). *Rotation for maximum association between factors and tests.* Unpublished manuscript, Biometric Laboratory, George Washington University, Washington, DC.

Mulaik, S. A. (2005). Looking back on the indeterminacy controversies in factor analysis. In A. Maydeu-Olivares & J. J. McArdle (Eds.), *Contemporary psychometrics: A festschrift for Roderick P. McDonald* (pp. 173–206). Mahwah, NJ: Erlbaum.

Muthén, L. K., & Muthén, B. O. (1998–2007). *Mplus user's guide.* Los Angeles: Author.

Russell, D. W. (2002). In search of underlying dimensions: The use (and abuse) of factor analysis in PSPB. *Personality and Social Psychology Bulletin, 28,* 1629–1646.

Spearman, C. (1904). "General intelligence," objectively determined and measured. *American Journal of Psychology, 15,* 201–293.

Steiger, J. H. (1979). Factor indeterminacy in the 1930's and the 1970's: Some interesting parallels. *Psychometrika, 44,* 157–166.

Steiger, J. H. (1994). SEPATH—A Statistica for Windows structural equations modeling program. In F. Faulbaum (Ed.), *Softstat '93: Advances in statistical software 4.* Stuttgart, Germany: Gustav Fischer Verlag.

Thompson, B., & Daniel, L. G. (1996). Factor analytic evidence for the construct validity of scores: A historical overview and some guidelines. *Educational and Psychological Measurement, 56,* 197–208.

Thurstone, L. L. (1947). *Multiple-factor analysis.* Chicago: The University of Chicago Press.

Yates, A. (1987). *Multivariate exploratory data analysis: A perspective on exploratory factor analysis.* Albany: State University of New York Press.